COSSAN TRAINING COURSE
on UNCERTAINTY QUANTIFICATION

Dealing with uncertainty

Edoardo Patelli

info@cossan.co.uk
www.cossan.co.uk

April 2020
Outline

1 Introduction
   - Motivation
   - Uncertainty?

2 Dealing with uncertainty
   - Method of analysis

3 Summary
Edoardo Patelli

Professor in risk and uncertainty quantification, University of Strathclyde, UK

- Project leader of the “COSSAN project”
- Head of Centre for Intelligent Infrastructure, Department of Civil and Environmental Engineering, University of Strathclyde
Modelling and Design

Digital twins, Virtual prototype

- Predictive mathematical and numerical models (e.g. FE) both in academia and industry
Computational Methods

Enable the “third and fourth paradigm” of science

Hardware and computational infrastructure largely available

Support research in the Digital Age

Need the development of better software and algorithms (in the service of science)
Challenges

High-consequence and safety-critical systems

- Increase confidence and consistency in safety predictions
- Modern standards require estimates of uncertainty
- Every disaster is unique
Uncertainty

Uncertainties appear everywhere! When using a mathematical model, careful attention must be given to uncertainties in the model. - Richard Feynman -

Uncertainty quantification is the missing piece of the puzzle in large scale computations. - Tim Barth -

We have to make the best model we possibly can, and then not trust it. - Robert Costanza -
Simulation standards require UQ
Increase confidence/consistency in safety predictions

From “Intuitive analysis” to “Quantitative analysis”

- **Most unusual variation** in the phenomena needs to be investigate (extreme case analysis)
- Looking for the “black swan”
Design steps

Traditional approach

Feasible Region
Design steps

Traditional approach
Design steps

Traditional approach

- Planning
- Preliminary design
- Design freeze
- Evaluation

Feasible Region
Design steps

Traditional approach
Design steps

Traditional approach

Diagram showing the design process with steps:
1. Planning
2. Preliminary design
3. Design freeze
4. Evaluation

Feasible Region: ✓
Uncertainties: 🔴
Design steps

Traditional approach
Design steps

Robust Design

Planning → Preliminary design → Design freeze → Evaluation

Uncertainties
Unavoidable uncertainties I

Irreducible (aleatory) uncertainties

- Parameters intrinsically uncertain
- Value varies at each experiment
- E.g.: future environmental conditions, stochastic processes

Sea Level Projections

- CGCM1
- CSIRO Mk2
- ECHAM4/OPYC3
- GFDL
- HadCM2
- DOE PCM
- MRI2

Observations

Sea Level Rise (cm)
Unavoidable uncertainties II

Reducible (epistemic) uncertainties

- Quantities that can in theory be precisely determined
- Practically they are not measured (e.g., too expensive, measurement destroys the item)
- Field properties, simplified model
Unavoidable uncertainties

Epistemic (reducible)

Aleatory (irreducible)

Physical

Modelling

Spectrum of uncertainties
How to deal with uncertainties?

- Effects of uncertainty need to be included at the design stage
- From “Intuitive analysis” to “Quantitative analysis”
How to deal with uncertainties?

- Effects of uncertainty need to be included at the design stage
- From “Intuitive analysis” to “Quantitative analysis”
How to deal with uncertainties?

- Effects of uncertainty need to be included at the design stage
- From “Intuitive analysis” to “Quantitative analysis”

Robust/Reliability-Based Optimisation

Deterministic Optimisation

Uncertainty → Model Analysis → Uncertainty Quantific. (failure probability)
How to deal with uncertainties?

- Effects of uncertainty need to be included at the design stage
- From “Intuitive analysis” to “Quantitative analysis”
How to deal with uncertainties?

- Effects of uncertainty need to be included at the design stage
- From “Intuitive analysis” to “Quantitative analysis”
Engineers are aware of parameter uncertainty

- Assume that a given degree of variability (e.g. 10%) on the input side results in a similar variability of the output (say 10%)
- Assumption is very optimistic; dangerous deviations from the nominal response remain unnoticed
- Robustness cannot be assessed with a deterministic approach
Goal uncertainty management

Safety factors are well established for design purposes
  • Do not necessarily reflect the actual response variability

Challenges
  • Models might involve several thousands of uncertain parameters
  • Important parameter(s) might be missed in case only a subset of parameters is investigated.
Goal uncertainty management

Safety factors are well established for design purposes
  - Do not necessarily reflect the actual response variability

Challenges
  - Models might involve several thousands of uncertain parameters
  - Important parameter(s) might be missed in case only a subset of parameters is investigated.
Goal uncertainty management
Increase confidence and consistency in safety predictions

The effect of uncertain parameters on critical response items need to be evaluated

Quantifying and managing uncertainty

- Improve design and solutions
- Improve model fidelity (more accountable forecasts)
- Better (natural and industrial) risk control
- Robust performance (resilient design)
- Risk informed decision
Stochastic analysis

Requirements

Efficient analysis requires:
- High Performance Computing
- Advanced simulation methods
  (Cossan software)

Computational modelling is the third pillar of scientific research
Stochastic analysis

Requirements

Efficient analysis requires:

- High Performance Computing
- Advanced simulation methods (Cossan software)

Computational modelling is the third pillar of scientific research
Outline

1. Introduction
   - Motivation
   - Uncertainty?

2. Dealing with uncertainty
   - Method of analysis

3. Summary
Dealing with uncertainty: Requirements

Quantifying

- Modelling and refinement of uncertainty based on experimental data, simulations and/or expert opinion
- Propagation of uncertainties through system models
- Parameter ranking and sensitivity analysis in the presence of uncertainty

Managing

- Identification of the parameters whose uncertainty is the most/least consequential
- Worst-case system performance assessment
- Design in the presence of uncertainty
Dealing with uncertainty: Requirements

Quantifying

- **Modelling and refinement** of uncertainty based on experimental data, simulations and/or expert opinion
- **Propagation** of uncertainties through system models
- Parameter ranking and **sensitivity analysis** in the presence of uncertainty

Managing

- Identification of the parameters whose uncertainty is the most/least consequential
- **Worst-case system performance assessment**
- **Design** in the presence of uncertainty
Questions to be answered I

- How are the uncertainties modelled?
- What is the variability of the quantities of interest?

- How does the uncertainty affect the performance of the model/system?
- Is the uncertainty of the prediction within acceptable bounds?
Questions to be answered I

- How are the uncertainties modelled?
- What is the variability of the quantities of interest?

⇒ Answers by uncertainty characterisation

- How does the uncertainty affect the performance of the model/system?
- Is the uncertainty of the prediction within acceptable bounds?
Questions to be answered I

- How are the uncertainties modelled?
- What is the variability of the quantities of interest?

⇒ **Answers by uncertainty characterisation**

- How does the uncertainty affect the performance of the model/system?
- Is the uncertainty of the prediction within acceptable bounds?

⇒ **Answers by uncertainty quantification**
Questions to be answered II

Which parameters are important?

- Which parameter should be assessed better to reduce the uncertainty of the prediction?
- Which parameters have little influence on the considered response?

⇒ Answers by Sensitivity Analysis
Questions to be answered II

Which parameters are important?
- Which parameter should be assessed better to reduce the uncertainty of the prediction?
- Which parameters have little influence on the considered response?

⇒ Answers by Sensitivity Analysis
Questions to be answered III

- Is the product robust?
- Is the product safe wrt the uncertainties?
- Is the product safe and optimized wrt the uncertainties?

⇒ Answers by Robust Design
⇒ Answers by Reliability Analysis
⇒ Answers by Reliability Based Optimization
Questions to be answered III

- Is the product robust?
- Is the product safe wrt the uncertainties?
- Is the product safe and optimized wrt the uncertainties?

⇒ Answers by Robust Design
⇒ Answers by Reliability Analysis
⇒ Answers by Reliability Based Optimization
Outline

1. Introduction
   - Motivation
   - Uncertainty?

2. Dealing with uncertainty
   - Method of analysis

3. Summary
Summary

- Needs of stochastic analysis
- Dealing with uncertainty

What next?

- Concepts of probability
- Design of experiment