Introduction

- PhD Student, 2nd Year of Study
- Supervisors: Professor Edoardo Patelli, and Associate Professor Alice Cicirello
- Project: Application of Bayesian Model Updating in Probabilistic Safety Assessment of Nuclear Power Plants
- Affiliated with Singapore Nuclear Research and Safety Initiatives (SNRSI)
- Research Interests:
  - Bayesian Model Updating;
  - Mathematical Modelling;
  - Nuclear Energy;
  - Predictive Maintenance;
  - Probabilistic Safety Assessment
Pre-requisites

There are no pre-requisites for this part of the course.
Overview

1. Introduction to Model Updating
2. Motivation behind Model Updating
3. Deterministic vs Probabilistic Model Updating
4. Introduction to Bayesian Model Updating
5. Brief Introduction to Advanced Sampling Techniques
Definition of the Forward problem:

\[ f(\theta) \]

via Monte-Carlo
Model Update
Conceptual Introduction and Motivation

Definition of the Inverse problem:

\[ f^{-1}(y) \]

The Answer: Model Updating
Model Update
Conceptual Introduction and Motivation

- Mathematical models are used to describe virtual behaviours of engineering structures under operational and extreme conditions;

- For the model to produce output representative of the structure’s response, there is a need to update the model’s input parameter(s);

- This seeks to minimise the difference between the model output and the measured response of the system.
Model Update

Conceptual Introduction and Motivation

Model Updating

Deterministic

Probabilistic
Deterministic Model Update

Conceptual Introduction

- Involves the calibration of model based on one data-set;
- Provides point estimates of $\theta$;
- Assumes one definite model to describe the data-set;
Deterministic Model Update

Conceptual Introduction

There are 4 main problems in performing Deterministic Model Update:

- Mathematical model is assumed to fully describe the Physics of the problem;
- Does not account the factor of “noise” that exists in measurements;
- Does not account for variation of the measured response of nominal identical structures even under the same loading/condition due to manufacturing and material variability;
- Point estimates may not represent the entire set of all possible solutions to $\theta$. 
Deterministic Model Update

Conceptual Introduction
Model Update
Conceptual Introduction and Motivation
Bayesian Model Update

Conceptual Introduction and Work-flow

A Probabilistic Model Updating technique based on Bayes’ Inference:

\[ P(\theta|D) = \frac{P(\theta) \cdot P(D|\theta)}{P(D)} \]  \hspace{1cm} (1)

where \( P(\theta) \) is the Prior; \( P(D|\theta) \) is the Likelihood function; \( P(\theta|D) \) is the Posterior; \( P(D) \) is the Evidence. \( \theta \) denotes vector of epistemic parameters; \( D \) denotes vector of observations.
Bayesian Model Update

Conceptual Introduction and Work-flow

\(P(D)\) is a normalisation factor which is independent of \(\theta\) and, thus, a numerical constant.

Equation (1) can therefore be re-expressed as shown below:

\[P(\theta|D) \propto P(\theta) \cdot P(D|\theta)\]
To generate samples from a distribution, a standard tool would be **Monte-Carlo** sampling;

HOWEVER...recall that:

\[ P(\theta|D) \propto P(\theta) \cdot P(D|\theta) \]

Standard Monte-Carlo technique is **unable** to sample from un-normalised distribution function;

We need advanced sampling techniques to do so ... ...
Bayesian Model Update
Techniques and Tools

Advanced Sampling Methods

- Markov Chain Monte-Carlo (MCMC)
- Sequential Monte-Carlo (SMC)
- Transitional Markov Chain Monte-Carlo (TMCMC)
Conclusion

Summary

- Overview of Model Updating concept and the need for it;
- Deterministic vs Probabilistic Model Updating;
- Understanding of Bayesian Model Updating concept;
- Familiarisation with Advanced Sampling Techniques and the need for them.
Conclusion

Follow-up

What's next

- Read Lecture Notes: Bayesian Model Updating [Part II]
- Watch Lecture Video: Bayesian Model Updating [Part II]